SPRING 1999 RELEASE ITEM
Grade 11 Mathematics

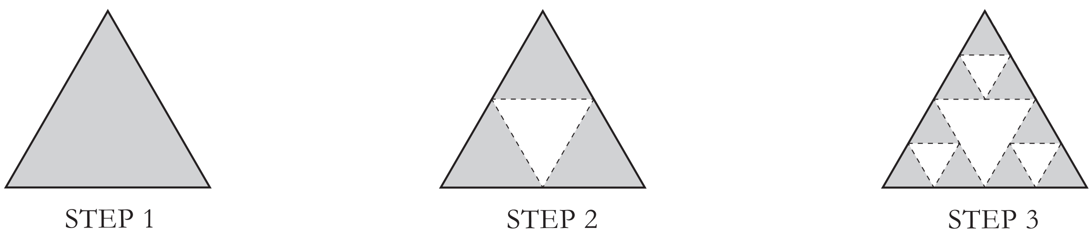
SAMPLE OPEN-RESPONSE QUESTION

How do students provide evidence of what they know and can do in mathematics?

The following is an example of an open-response question designed to provide an opportunity for students to show what they know and can do in the area of mathematics:

Triangle Patterns

Use the diagrams below to answer the question.



The diagrams above show the first three steps of a process used to make a triangle pattern. In Step 1, the large triangle has an area equal to 1 unit. In Step 2, the large triangle is divided into 4 congruent triangles, and the middle triangle is removed.

a. What is the area of the **shaded** region in Step 2?

In Step 3, the dividing process continues. Each of the shaded triangles in Step 2 is divided into 4 congruent triangles. Each middle triangle is removed, and the shaded parts remain. The dividing process continues until the total shaded area is less than $\frac{1}{2}$ unit.

b. How many steps does it take until the **shaded** area is less than $\frac{1}{2}$ unit? Give a complete explanation to justify your answer.

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MATHEMATICS CONTENT

What is the relationship of the assessment to the curriculum?

The content of the open-response question "Triangle Patterns" addresses the following Mathematics Academic Expectations: "Students use mathematical ideas and procedures to communicate, reason, and solve problems" (1.5-1.9); "Students understand number concepts and use numbers appropriately and accurately" (2.7); "Students understand various mathematical procedures and use them appropriately and accurately" (2.8); "Students understand space and dimensionality concepts and use them appropriately and accurately" (2.9); "Students understand measurement concepts and use measurement appropriately and accurately" (2.10); "Students understand mathematical change concepts and use them appropriately and accurately" (2.11); and "Students understand mathematical structure concepts including the properties and logic of various mathematical systems" (2.12).

This question provides a way for students to show their understanding of several concepts from the *Core Content for Mathematics Assessment*. In addition to the general concepts of area and congruence, students are asked to show their understanding of computation with rational numbers (i.e., fractions); equivalence and order relations; pattern sequences involving area and congruence; and relationships between geometric concepts and algebraic procedures.

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PERFORMANCE EXPECTATIONS

How good is good enough?

An appropriate student response should provide evidence of the student's ability to understand congruence, area, and pattern sequences, and to determine the areas of fractional parts of a figure by correctly performing a mathematical operation (i.e., multiplication) with fractions.

For example, an appropriate response to this question will show that the student can

- understand the concept of congruent triangles;
- understand the triangle pattern sequence shown in the three steps of the diagram (i.e., a large triangle is divided into four smaller congruent triangles with the middle triangle removed, and then each of the three remaining smaller triangles are further divided into four smaller congruent triangles);
- accurately compute the area (in units) of the fractional part of the large triangle that is shaded as shown in Step 2 (i.e., the area of three of the four smaller congruent triangles making up the large triangle);
- accurately compute the area (in units) of the fractional part of the large triangle that is shaded as shown in Step 3;
- accurately determine the correct number of steps needed to continue the pattern until the shaded area is less than $\frac{1}{2}$ unit (i.e., accurately determine the 4th term of the pattern sequence); and
- accurately and clearly explain the procedure used to determine the correct number of steps needed.

Successful student work should provide convincing evidence that the student can use mathematical knowledge to address the relevant issue(s), although the response may not address all details and may contain minor mathematical errors.

APPLICATIONS

How is this relevant?

By successfully answering this question, students demonstrate the ability to recognize a mathematical pattern sequence, to represent the pattern sequence with rational expressions (i.e., fractions), and to accurately compute the area of part of a geometric figure using these rational expressions (i.e., fractions). The ability to recognize a mathematical pattern sequence and to represent the pattern sequence with rational expressions (be they fractions or other expressions) may be useful to students in school and in adult life in a variety of subject areas and fields (e.g., math, science, psychology, economics, architecture). Also, the ability to accurately compute the area of part of a figure may be useful to students in a variety of real-world tasks (e.g., determining the amount of carpet needed to cover part of a floor that has an irregular shape; determining the amount of tiles needed to tile a wall in a geometric design).

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SAMPLE OPEN-RESPONSE QUESTION

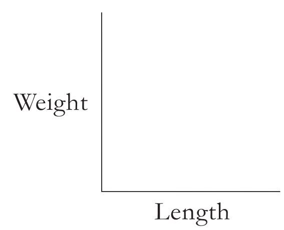
How do students provide evidence of what they know and can do in mathematics?

The following is an example of an open-response question designed to provide an opportunity for students to show what they know and can do in the area of mathematics:

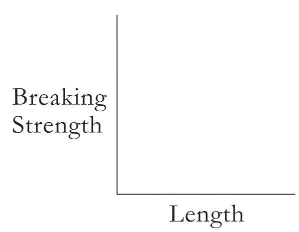
Rope Strength

Manufacturers consider several different features when making rope for various uses. In this problem, you will graph or find a relationship between these different features. (To answer this question, you will need to draw 3 graphs in your Student Response Booklet.)

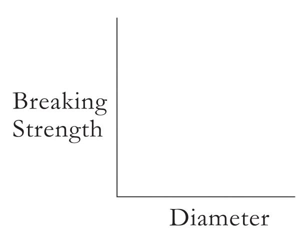
a. In your Student Response Booklet, copy the labeled axes below and sketch a graph showing that if you double the length of a piece of rope, you double its weight.



b. In your Student Response Booklet, copy the labeled axes below and sketch a graph showing that the breaking strength of a rope is not affected by its length.



c. In your Student Response Booklet, copy the labeled axes below and sketch a graph showing that if you double the diameter of a rope, you multiply its breaking strength by 4.



SAMPLE OPEN-RESPONSE QUESTION (cont.)

d. The table of data below shows the relationship between the diameter of a rope and its breaking strength. Using the data, write a formula for the breaking strength (S) in terms of the diameter (d).

diameter (in mm)	0	2	4	6	8	10	12
approximate breaking strength (in kg)	0	80	320	720	1280	2000	2880

BE SURE TO LABEL YOUR ANSWERS (a), (b), (c), AND (d).

MATHEMATICS CONTENT

What is the relationship of the assessment to the curriculum?

The content of the open-response question "Rope Strength" addresses the following Mathematics Academic Expectations: "Students use mathematical ideas and procedures to communicate, reason, and solve problems" (1.5-1.9); "Students understand number concepts and use numbers appropriately and accurately" (2.7); "Students understand various mathematical procedures and use them appropriately and accurately" (2.8); "Students understand mathematical change concepts and use them appropriately and accurately" (2.11); and "Students understand mathematical structure concepts including the properties and logic of various mathematical systems" (2.12).

This question provides a way for students to show their understanding of several concepts from the *Core Content for Mathematics Assessment*. In addition to the general concept of linear versus exponential functions, students are asked to show their understanding of ratio measures such as slope, the construction of graphs to match given situations, and the relationship between a given situation, a graph, and an algebraic equation.

PERFORMANCE EXPECTATIONS

How good is good enough?

An appropriate student response should provide evidence of the student's understanding of linear and exponential relationships, and of the ability to translate various situations into linear or exponential graphs and algebraic expressions.

For example, an appropriate response to this question will show that the student can

- accurately translate a given situation—that is, a piece of rope doubles in weight as its length doubles—into a linear graph that reflects this situation (i.e., the graph has a positive slope);
- accurately translate a given situation—that is, the breaking strength of a rope is not affected by its length—into a linear graph that reflects this situation (i.e., the graph has zero slope);
- accurately translate a given situation—that is, the doubling of a rope's diameter multiplies its breaking strength by 4—into an exponential graph that reflects this situation (i.e., the graph has a rising concave curve);
- accurately interpret data displayed in table form; and
- accurately write a formula (i.e., an exponential function) based on the data in the table.

Successful student work should provide convincing evidence that the student can use mathematical knowledge to address the relevant issue(s), although the response may not address all details and may contain minor mathematical errors.

APPLICATIONS

How is this relevant?

This problem addresses the relationships of real-life phenomena to both linear and exponential models. By successfully answering this question, students demonstrate an ability to translate given situations into linear or exponential graphs and to translate data displayed in table form into an exponential function. The ability to model situations and data both graphically and algebraically will help students to better understand and make use of mathematical symbols as they relate to real life.

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SAMPLE OPEN-RESPONSE QUESTION

How do students provide evidence of what they know and can do in mathematics?

The following is an example of an open-response question designed to provide an opportunity for students to show what they know and can do in the area of mathematics:

Predictable Data

The faces of a six-sided cube are printed with the numbers 1 through 6. Chris performed an experiment by rolling the cube 60 times, recording the results after each roll. The results are as follows:

Number rolled	1	2	3	4	5	6
Frequency	13	9	10	7	12	9

Chris's friend says that the results cannot be correct because their teacher said that the probability of rolling any one of the six outcomes is $\frac{1}{6}$.

a. Based on the teacher's comment, what results should Chris theoretically expect to see after 60 rolls of the six-sided number cube? Copy the chart below into your Student Response Booklet, and then complete the chart with the theoretically expected values.

Number rolled	1	2	3	4	5	6
Frequency						

b. Do you agree or disagree with Chris's friend that the original results cannot be correct? Explain your reasoning.

MATHEMATICS CONTENT

What is the relationship of the assessment to the curriculum?

The content of the open-response question "Predictable Data" addresses the following Mathematics Academic Expectations: "Students use mathematical ideas and procedures to communicate, reason, and solve problems" (1.5-1.9); "Students understand number concepts and use numbers appropriately and accurately" (2.7); "Students understand various mathematical procedures and use them appropriately and accurately" (2.8); and "Students understand and appropriately use statistics and probability" (2.13).

This question provides a way for students to show their understanding of several concepts from the *Core Content for Mathematics Assessment*. In addition to the general concept of probability, students are asked to demonstrate their understanding of computation with rational values (i.e., fractions) and of how sample size can influence results.

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PERFORMANCE EXPECTATIONS

How good is good enough?

An appropriate student response should provide evidence both of the student's ability to use probability to compute theoretically expected results and of the student's understanding of the relationship between experimental and theoretical probability and the effects of sample size on actual results.

For example, an appropriate response to this question will show that the student can

- accurately use a given probability (i.e., $\frac{1}{6}$) to compute theoretically expected results for 60 rolls of a six-sided number cube;
- accurately complete a chart that shows the theoretically expected results for 60 rolls of a six-sided number cube;
- correctly interpret the results of Chris's experiment with a six-sided number cube (i.e., clearly state disagreement with Chris's friend who says that Chris's original results for 60 rolls of a six-sided number cube cannot be correct); and
- clearly explain the reason for the disagreement by referring to the small sample size of the experiment that produced the original results.

Successful student work should provide convincing evidence that the student can use mathematical knowledge to address the relevant issue(s), although the response may not address all details and may contain minor mathematical errors.

APPLICATIONS

How is this relevant?

This question addresses the importance of probability situations. By successfully answering this question, students demonstrate an ability to use a given probability to compute the expected results of an experiment or situation and to recognize the relative importance of sample size in determining the validity of an experiment's or situation's actual results. This ability may be useful to students both in school (e.g., in science or math classes) and in adult life (e.g., in the analysis of probability situations that they encounter or read about).

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SAMPLE OPEN-RESPONSE QUESTION

How do students provide evidence of what they know and can do in mathematics?

The following is an example of an open-response question designed to provide an opportunity for students to show what they know and can do in the area of mathematics:

Storage Tank Measurement

The Mattox Oil Company has a 90-foot-tall cylindrical storage tank that contains 210,000 gallons of oil when it is full.

a. Find the radius of the tank. Show your work.

($V = \pi r^2 h$ and 1 gallon = 0.1337 cubic feet)

b. How tall would the tank in **part a** have to be in order to double the volume? Show your work.

c. How would the volume of the tank in **part a** be affected if both the radius and height of the tank were doubled? Explain your answer.

MATHEMATICS CONTENT

What is the relationship of the assessment to the curriculum?

The content of the open-response question "Storage Tank Measurement" addresses the following Mathematics Academic Expectations: "Students use mathematical ideas and procedures to communicate, reason, and solve problems" (1.5-1.9); "Students understand number concepts and use numbers appropriately and accurately" (2.7); "Students understand various mathematical procedures and use them appropriately and accurately" (2.8); "Students understand space and dimensionality concepts and use them appropriately and accurately" (2.9); "Students understand measurement concepts and use measurement appropriately and accurately" (2.10); and "Students understand mathematical change concepts and use them appropriately and accurately" (2.11).

This question provides a way for students to show their understanding of several concepts from the *Core Content for Mathematics Assessment*. In addition to the general concept of how numbers in the real number system relate to each other, students are asked to show their understanding of algebraic equations involving irrational numbers (i.e., square roots and π), equivalence (i.e., conversion between measurement systems), variables and constants in an equation, and the connection between algebraic procedures and geometric concepts.

PERFORMANCE EXPECTATIONS

How good is good enough?

An appropriate student response should provide evidence of the student's understanding of how to solve algebraic equations and how these equations relate to geometric concepts.

For example, an appropriate response to this question will show that the student can

- clearly understand a set of given formulas (i.e., the formula for volume and the formula for converting gallons into cubic feet);
- use the appropriate formula to accurately convert the number of gallons in a tank into cubic feet;
- correctly set up an algebraic equation involving irrational numbers to model a given geometric context (i.e., replace the variables in the formula for volume with the appropriate numbers given in the problem);
- accurately solve the algebraic equation to find the radius of the tank;
- correctly set up and solve an algebraic equation involving irrational numbers to determine how tall the tank would have to be in order to double its volume;
- clearly and accurately explain how the volume of the tank would be affected if both the radius and height of the tank were doubled; and
- clearly and accurately explain the procedure (i.e., the algebraic equation) used to determine how the volume of the tank would be affected if both the radius and height of the tank were doubled.

Successful student work should provide convincing evidence that the student can use mathematical knowledge to address the relevant issue(s), although the response may not address all details and may contain minor mathematical errors.

APPLICATIONS

How is this relevant?

This problem addresses the importance and utility of algebraic equations to solve real-life phenomena. By successfully solving this problem, students demonstrate an understanding of how algebraic equations relate to geometric concepts and an ability to solve equations involving irrational numbers. This knowledge and ability will help students to better understand and make use of mathematical symbols (i.e., algebraic equations) as they relate to real-life situations.